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ARE PARTICULAR ABILITIES NECESSARY FOR THE PUPIL TO GAIN AN UNDERSTANDING OF THE ELEMENTARY AND SECONDARY MATHEMATICS AS USUALLY GIVEN AT THE PRESENT TIME?

By Maurice J. Babb.

The director of one of our most prominent laboratories of anatomical research told me during this last month that he has been having a frequently recurring and vivid dream for a year or more, to the effect always, that somehow he had failed to make good in mathematics, and that discovery was both imminent and inevitable. He confessed the sense of guilt and horror of exposure was so real as to be depressing even after he awoke. I have been wondering whether he is a sort of vicarious sufferer for the sins of his profession in particular and of this age in general. It is good to find some evidence of the arousal of a mathematical conscience. Simultaneously the much-maligned appendix vermiformis has been found to have unique and valuable functions, to the great consternation of numerous and fashionable people. All this is personally particularly interesting, as my first visit to the University of Pennsylvania was as a prospective medical student and naturally I drifted into the laboratory of the late Professor John Rider, whom you doubtless remember was rejected in his youth as unable to appreciate and absorb the knowledge as then dispensed by the adjacent public schools. He told me that he recently had been a student along with the sophomores in calculus in order to gain sufficient knowledge of the mathematics to be able to express the cellular development of the ear from the brain in embryo mice. This year a student of my calculus class comes in the same way that he may gain knowledge which he may apply to his researches on muscle fibers. These men were said not to have had mathematical minds in their school days. Professor David Eugene Smith has recently given the New England branch of this Society a list of seemingly hopeless mathematical outcasts which on further examination has turned out to be a list of the great men of the science.

In early days one was a mathematician having been first a philosopher. And methinks, as I scan the pages of mathematical history, that this was true of practically all who have really extended the science.

When I turn to Poincare whose recent death deprived us of our greatest modern mathematician, I find, "The very possibility of the science of mathematics seems an insoluble contradiction. If this science is deductive only in appearance, whence does it derive that perfect rigor no one dreams of doubting? If on the contrary, all propositions it enunciates can be deduced one from another by the rules of formal logic, why is not mathematics reduced to an immense tautology?"*

For years, much more so since the Civil War, our elementary mathematics was taught by preachers who had strained their voices, and others, with here and there a thoughtful soul who appreciated the subject, knew something of real mathematical rigor, yet thought it no crime to "pass over rapidly, but without skipping stations, the road traversed slowly by the founders of the science." Poincare later asks, "Why is so long a preparation necessary to become habituated to this perfect rigor, which, it seems, should naturally impress itself on all good minds?" This is a logical and psychological problem well worthy of study, but we shall not take it up.

A centipede was happy quite
Until a toad in fun
Said, "Which leg moves after which?"
This troubled him to such a pitch
He fell exhausted in a ditch,
Not knowing how to run.

Suffice it to say that mathematics is a sequence subject in more ways than one and that there is a mathematical dialect. Blessed is the teacher who is student enough to appreciate more and more the beauties of its rigor and still remember the time when he would have deemed these same beauties irksome subtleties. Such a teacher will not forget that a real leader rides not too far ahead. All capable investigators are usually clearer of statement than their satellites and most of us will already agree that particular abilities are not necessary for the pupil to gain an understanding of the elementary and secondary mathematics as the average teacher understands them.

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* "Science and Hypothesis."

Until education became so general the comparatively few privileged pupils partook in all meekness the course prescribed for them. If there was not a place for the finished product, the community tolerated it as the Indians did those deemed similarly misfortunate. It is only when the mass moves up a stage that new adjustments are needed.

Probably in no science was there less concern about pedagogy than in mathematics. To-day we are asking, "Can literature be taught?" If the same question were asked about mathematics, what would be your answer? In my own experience as a teacher and private tutor I have yet to find an individual who failed to appreciate and enjoy mathematical reasoning when the simple underlying principles were made definitely clear. To do this needed both an understanding of the problem and its setting, both in the subject and in the mind of the pupil.

Most of our opinions to-day are founded on our limited personal observation. That personal observations are somewhat narrow is evinced by the following from over in Salisbury, N. H. The school committee was holding a meeting to consider the purchase of new text-books. An old man with a wabble in his chin and a tremble in his voice arose and said: "Didn't we study out of those books? Ain't what was good enough for us good enough for you? What's the good of book-learnin' anyway? We need something practical. I mind the time when I used to come along the old North road and see Dan Webster up in his gable window with a lamp aburnin' and him writin' and studyin' out o' his books as late as nine o'clock at night. What did it all amount to? Why he went away from here and we never heard of him afterwards."

Recently some investigators, notably Stone and Courtis and Thorndyke, with whose work you are familiar, have further simplified their test and endeavored to cover a wide field of observation. Our own arithmetic committee has meant to add considerably to our knowledge but will probably have to call upon well-equipped research laboratories and it will be some time before reliable deductions will be in the hands of the general teaching body. The handling of statistics is a subject on which I do not wish to speak at present. A very interesting recent investigation by E. O. Lewis attempts to find the most popular and the most unpopular subject by asking questions of 8,000 boys and girls in London and in South Wales. Some of the reasons are interesting; many are vague. But preference generally is clear-cut and the same opinions after as before the long vacation. The comment on grammar is: "I don't like grammar because I am not good at guessing." Such a remark is as pathetic as it is true. Scripture, which is the only subject to precede mathematics in the order of the day in England, was dismissed with: "Scripture will be no use to me after I am fourteen." One said: "Arithmetic prevents the shop-keeper from cheating us." Another said: "I don't like arithmetic, because there is no fun in it." In all the papers arithmetic was placed either very high or very low. So in the general average it occupies a position slightly above the middle. Here the average is not significant. An examination of the grouping seems to bear out what we have hinted before, namely, that where the underlying principles are well understood, mathematics is pleasing. But where these are not thoroughly clear the very discipline of the subject makes the pupil disgusted with the futility of his parody on reasoning.

It is said to be proverbial in America to tell what we intend to do in the next five years, whereas the German under the same conditions reviews what he has done during the last five years. Probably another paper may deduce from this fact why we do less mathematics in two and one tenth school years than the Germans do in one and two tenths. What causes the reversal of the digits? A popular American institute speaker once confessed to me his secret and gave me the following recipe. He said, "Choose a subject about which you know nothing and about which you are reasonably sure your audience knows nothing. Then you will not only have full freedom for any flight of fancy but you can easily flatter your hearers by your assumption of their familiarity with the subjects treated and with no danger of their exposing you, as they probably will be afraid to confer except in generalities." Many subjects in our curriculum may be placed in similar categories. But in mathematics the flow of eloquence is subject to disconcerting questions and checks. In many cases the sociologist cannot be satisfactorily contradicted until long after his epitaph has disintegrated, for no one knows why the wildest dreams may not come true in this wonderful world of ours. The reasoning in mathematics is obvious and checks are possible. It is the simplest sort of reasoning known. Mathematics has always been a science,—crude at first, so as not to harrass beginners. As one reaches more maturity, of course the primitive mathematical notions will undergo elaboration so profound that it becomes difficult to analyze them. Yet there is a gradual growth of breadth of vision. In other subjects the reasoning is contingent, this and that have to be weighed and balanced. The skilled chess player may see four or five moves ahead. He wins if he can see more than his opponent. In chess the moves are limited. If the moves are unlimited how much better is one who sees four particular moves than one who sees three? The younger the subject, the less exact the data, the less probable is exact diagnosis. The guess of the tyro may ring as true to the mob as the brief of the expert.

Is it not probable that the objection to mathematics is that like the Spartan we object to being found out, to finding ourselves out and having this fact shamelessly confronting us?

"The elements of mathematics usually taught," says that modern man, Augustus de Morgan, "are contained in the sciences of arithmetic, algebra, geometry and trigonometry; we have used these four divisions because they are generally adopted, though in fact algebra and geometry are the only two of them which are really distinct." Let us examine these subjects taught in slight detail.

In arithmetic expertness in the four fundamental rules depends upon continued practice. Much of greatest common divisor and least common multiple is no longer needed, since the introduction of decimals. Fractions offer the first real difficulty to the individual as they did to the race. I went to school with a boy who although he is now a builder of repute has never ventured to get on familiar terms with fractions. A great philosopher was Mother Goose! Here a teacher who knows more about fractions than he will teach his class and in so doing understands his different pupils' peculiar points of view will straighten out each mind, and extend the primitive ideas. The sequence of signs + — × ÷ is pure definition. The decimal point can be corralled if trained early by a logical rule such as only computers use. If denominate numbers are not simple, with but a single reduction, it is probably a sign of particular ability if the pupil rebels. Carpeting, paper-hanging and a lot more so-called practical problems, are not worked as in the arithmetic in practice. They belong to special trades whose usage it may in some cases be desirable for the teacher to learn from local tradesmen and suggest such inquiries subsequently to his pupils as a practical field of mathematical research. Business arithmetic resembles very slightly the type found in books and had also better be replaced by local problems. A friend of mine who settles estates and transfers property for a large trust company tells me he always finds that people have the transactions figured out to within a few cents, everybody in his own way, of course, but all sufficiently exact to prevent being cheated to any great extent. They realize when dealing with their own money the absurdity of the question of the young clerk in the banking office when his employer took him to task for making a mistake of a thousand dollars against the firm in calculating some exchanges. The boy's answer was: "Wasn't the principle correct?"

In all this work much of the tendency to errors common in our department stores, phone exchanges and general clerical work is due to too much paper in the schools, especially scrap paper. It is probable if our schools would teach pupils to hand in their work as originally entered and generally in ink, from the first grade up, that the next generation would show more improvement than many previous periods. If added to this the German method of chalk and talk were used, mathematical shorthand would take its proper place and oral English would be greatly improved. We used to have mental arithmetic till it became unreal. Then we figured and figured and wrote out long statement forms. What we want now is team work with chalk and talk. With this, transition to algebra will come easier. Written mathematics should be an obviously needed and suggestive shorthand for problems not easily rendered orally. Particular ability of the pupil is not needed here but better judgment on the part of the teacher and text-book maker. Even with an inferior text-book the chief trouble is in judgment on what to retain. Logarithms are about the only really excellent thing subjectively excluded from the arithmetic course. With proper training in arithmetic, algebra is but a more complete and generalized system of shorthand,—563 is $5(10)^2 + 6(10) + 3$ and not particularly different in conception from $5x^2 + 6x + 3$. Here again if the teacher is familiar with the history of the race's difficulties he can guide his pupils past blind alleys. Always he should remember that it is desirable for him to know much more than his pupils, and also it is even desirable in many cases that his pupils should not be taught as yet as much as he knows about particular subjects. Many teachers run to seed with too many kinds of factoring and then omit the factor theorem altogether. Let us here, as in arithmetic, also avoid the errors that come from mental tire caused by too long continued concentration. We should choose a few well-selected methods, always keeping in mind the significance of the result rather than the trick of guessing it. For example, very few pupils are taught what the root of an equation is. When principles are clear, complications may be introduced, but efficiency in manipulation of symbols is always a matter of practice. It cannot be retained inert, but seems easily regained. In radicals and imaginary numbers again the teacher must assure himself that he understands what he is doing and convince himself that he is not trying to transfer a habit confirmed by years to youthful minds in so many weeks. It is well to understand the principle of permanence, or of no exception.* Completing the square can be more mechanical than the formula. If proper drill has been had on roots the theory of quadratics will be enlightening and will make an impression for future use in analytical geometry. Simultaneous quadratics need geometric interpretation to show how special a case they are, depending entirely on recognition of forms. Arithmetic needs co-ordination here and continual review can be had in verifications and substitutions in formulæ. Operations used in arithmetic should be recognized, generalized and extended. Here again the teacher should not forget that he ought to know more than his pupils and not be proud of it. Why shouldn't he? Somewhere in here comes the geometry, making use of much of the algebra and supplying problems for the algebra, thus early showing that there are obverse and reverse sides of mathematics.

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* Schubert, "Mathematical Recreations and Essays."

Strange to say, geometry often seems to give more trouble to really good inquiring minds than to mediocre memorizers and dull acquiescent brains. The tacit hypotheses distributed all through the geometries we use do not agree with the pupil's particular manner of reasoning and he is confused and sometimes righteously belligerent. The English student attacks his ancient enemy Euclid single-handed, but with this hand in that of his tutor who knows his quality of selected mind. Here, too, after the geometric habit is formed, more theorems should become exercises and special cases generalized. All through mathematics too many things are committed to memory which would logically follow without effort. The pencil is used too little on the set proofs and too much on the problems.

Trigonometry does not consist merely in solving triangles. Indeed this is a minor part. The symbols of arithmetic and algebra are again combined with an added set of operators.

Let us then have less memory, more team work of chalk and talk and more cultivation of imagination and observation in our mathematics

"No," complained the Scotch professor to his students, "ye dinna use yer faculties of observation. For instance," picking up a jar of chemicals of vile odor, he stuck first a finger into it and then put his finger into his mouth, "taste it yourselves," he commanded. After each had licked his fingers and had felt rebellion through his whole soul, the professor exclaimed triumphantly, "I told ye so. Ye dinna use your faculties, for if ye had observed ye woud ha seen that the finger I stuck into the jar was nae the finger I stuck in my mouth."

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